Omitted Labels Induce Nontransitive Paradoxes in Causality

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POSTER PAPER

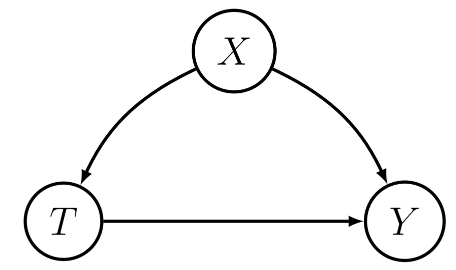
Computing Average Treatment Effect

Original Data:

T	X	Y 👎	Y 👍	Total
🚫	🤒	3	7	10
	🤮	1	0	1
💊	🤒	0	1	1
	🤮	7	3	10

IPW Data:

T	X	Y 👎	Y 👍	Total
🚫	🤒	3	7	10
	🤮	$\mathbf{10} \times 1$	$\mathbf{10} \times 0$	$\mathbf{10} \times 1$
💊	🤒	$\mathbf{10} \times 0$	$\mathbf{10} \times 1$	$\mathbf{10} \times 1$
	🤮	7	3	10



$$\Pr(👍 \mid do(💊)) = \sum_x \Pr(x)\Pr(👍 \mid x, 💊) = \frac{1/1 + 3/10}{2} = 13/20$$

$$\text{ATE} = \Pr(👍 \mid do(💊)) - \Pr(👍 \mid do(🚫)) = 3/10$$

Backdoor Adjustment (Pearl, 2009) Inverse Propensity Weighting (Horvitz and Thompson, 1952)

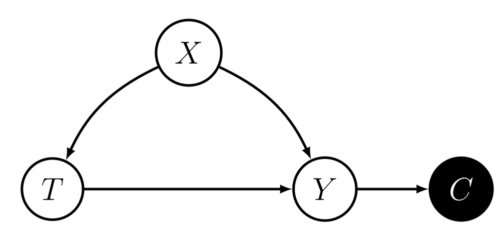
Simpon's Paradox from Omitted Labels

Original Data:

T	X	Y 👎	Y 👍	Y 🪦	Total
🚫	🤒	3	7	0	10
	🤮	1	0	99	100
💊	🤒	0	1	99	100
	🤮	7	3	0	10

IPW Weighted Data:

T	X	Y 👎	Y 👍	Y 🪦	Total
🚫	🤒	$\mathbf{10} \times 3$	$\mathbf{10} \times 7$	$\mathbf{10} \times 0$	$\mathbf{10} \times 10$
	🤮	1	0	99	100
💊	🤒	0	1	99	100
	🤮	$\mathbf{10} \times 7$	$\mathbf{10} \times 3$	$\mathbf{10} \times 0$	$\mathbf{10} \times 10$



$$\text{True ATE} = \frac{1/100 + 3/10}{2} - \frac{7/10 + 0/100}{2} = -19.5\%$$

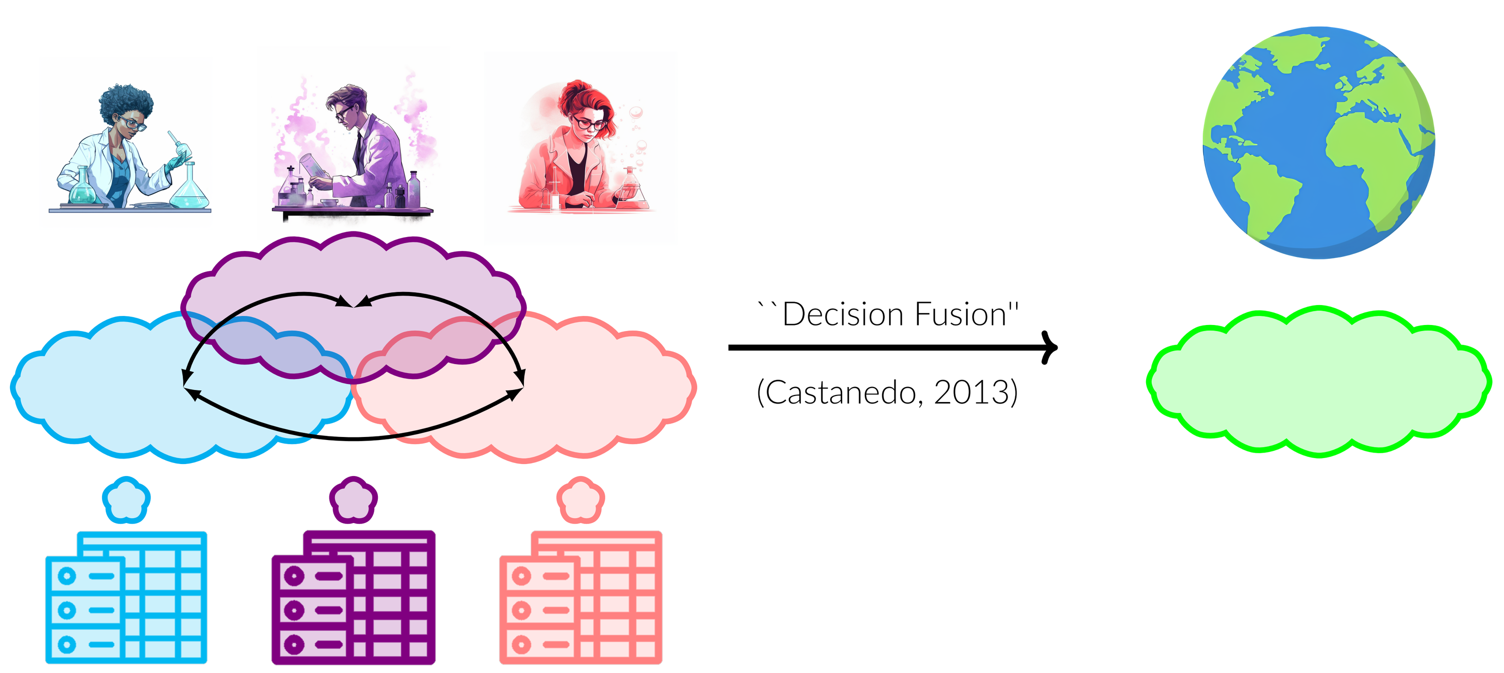
Context ATE $= \frac{31}{101} - \frac{70}{101} \approx -39\%$

Context ATE $= \frac{13}{20} - \frac{7}{20} = +30\%$

Networks of Experts

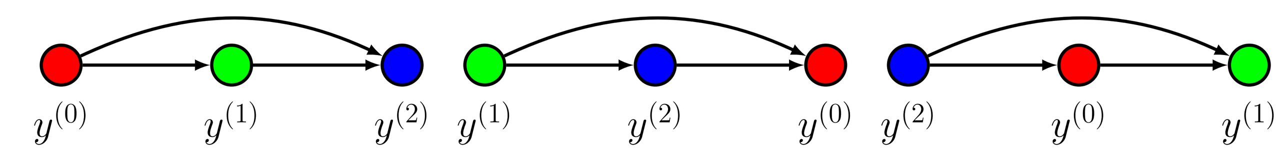
Data Contexts

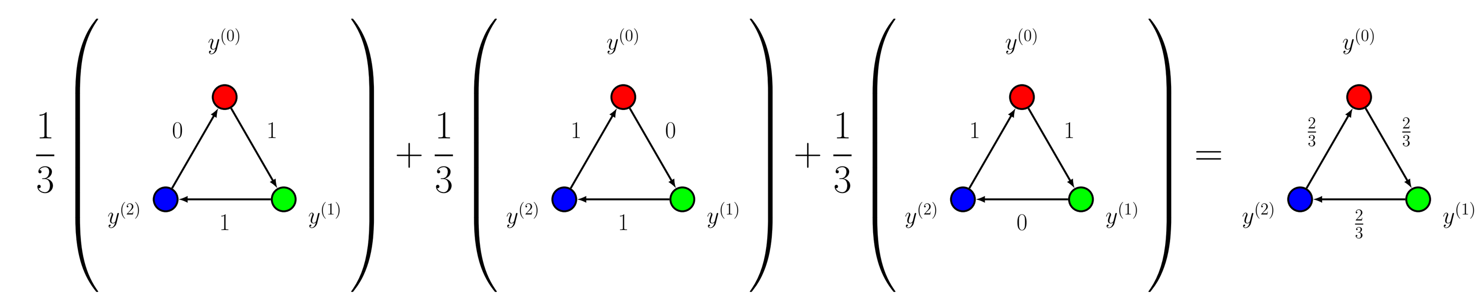
Target Context



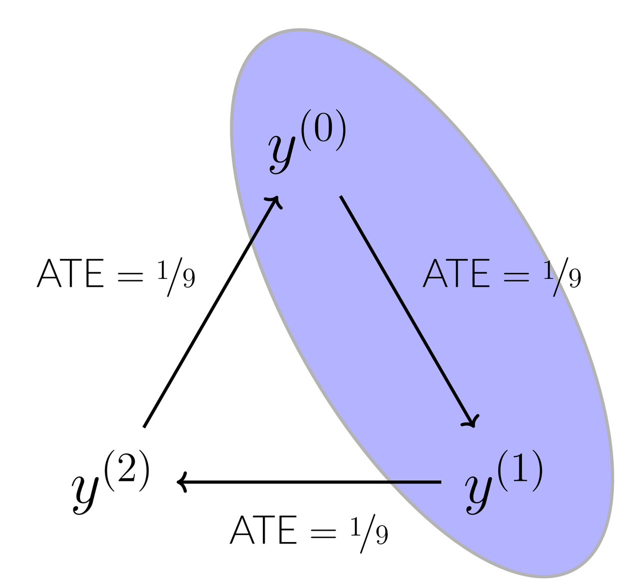
Condorcet Paradox in Voting (Nicolas et al., 1785)

Ranking $x^{(0)}$: Ranking $x^{(1)}$: Ranking $x^{(2)}$:

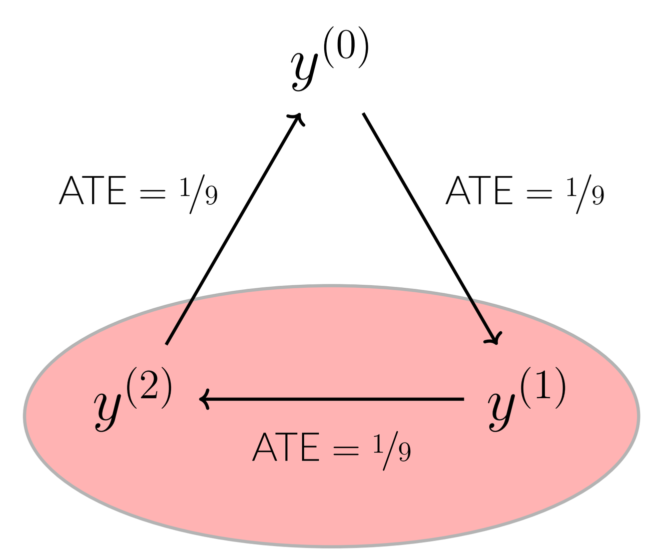




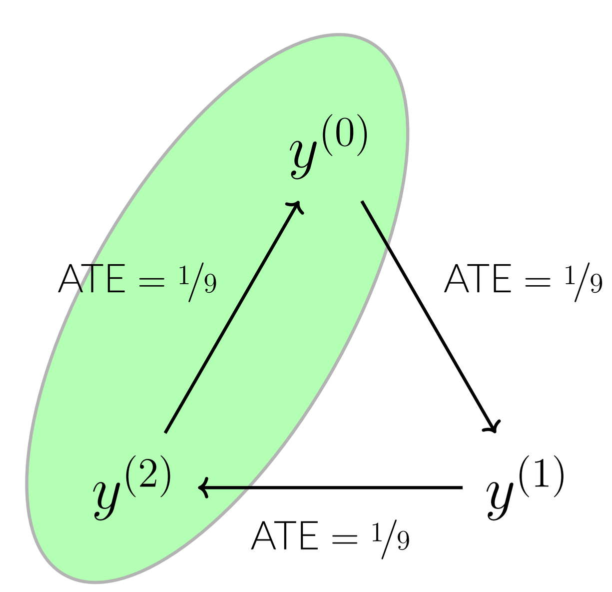
Condorcet Paradox in Causality



T	X	Y $y^{(0)}$	Y $y^{(1)}$	Y $y^{(2)}$
🚫	$x^{(0)}$	0	1	2
	$x^{(1)}$	2	0	1
	$x^{(2)}$	1	2	0
💊	$x^{(0)}$	2	1	0
	$x^{(1)}$	0	2	1
	$x^{(2)}$	1	0	2

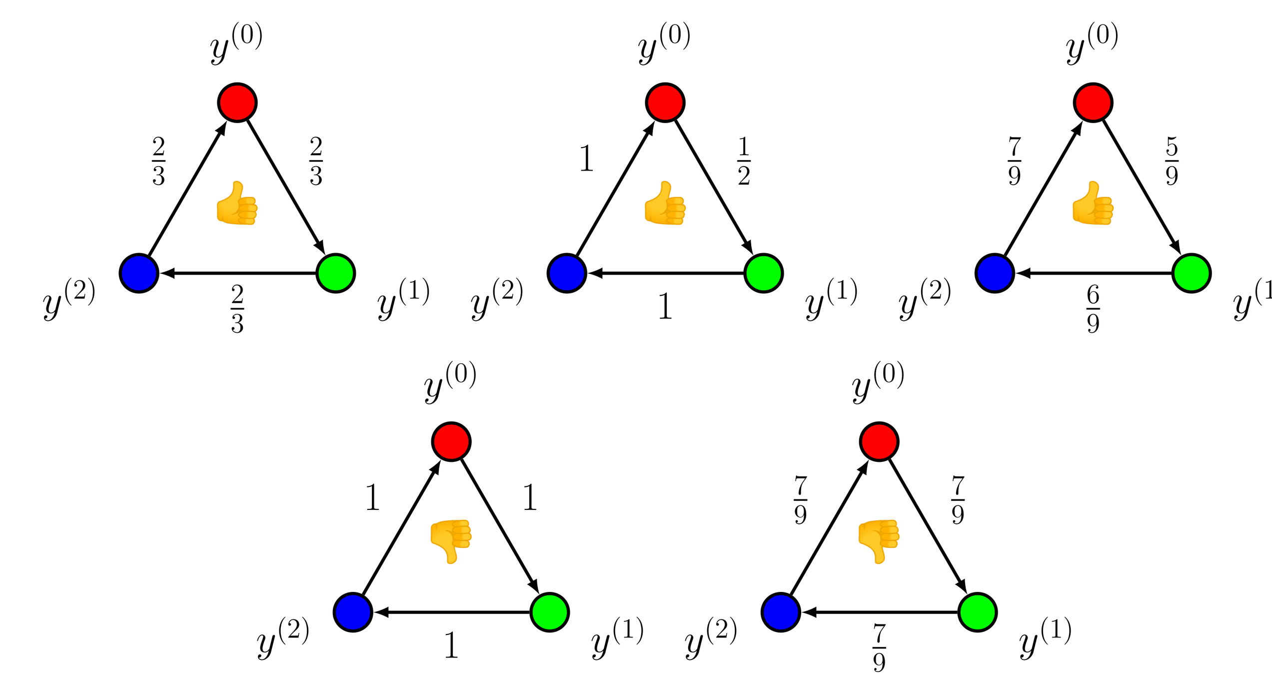


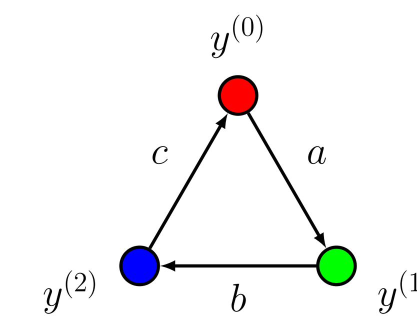
T	X	Y $y^{(0)}$	Y $y^{(1)}$	Y $y^{(2)}$
🚫	$x^{(0)}$	0	1	2
	$x^{(1)}$	2	0	1
	$x^{(2)}$	1	2	0
💊	$x^{(0)}$	2	1	0
	$x^{(1)}$	0	2	1
	$x^{(2)}$	1	0	2



T	X	Y $y^{(0)}$	Y $y^{(1)}$	Y $y^{(2)}$
🚫	$x^{(0)}$	0	1	2
	$x^{(1)}$	2	0	1
	$x^{(2)}$	1	2	0
💊	$x^{(0)}$	2	1	0
	$x^{(1)}$	0	2	1
	$x^{(2)}$	1	0	2

The Linear Ordering Polytope





$$1 \le a + b + c \le 2$$

"Triangle inequality" (Fishburn, 1992), or "curl condition" (Mazaheri, Jain, and Bruck, 2021)

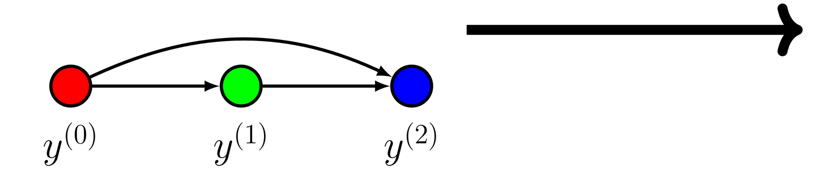
Equivalence

Theorem (Informal)

Networks of potential outcomes calculated relative to two labels have the same structure as the linear ordering polytope.

Individual voter preferences can be asymptotically ($\varepsilon \to 0$) approximated as a row of a table.

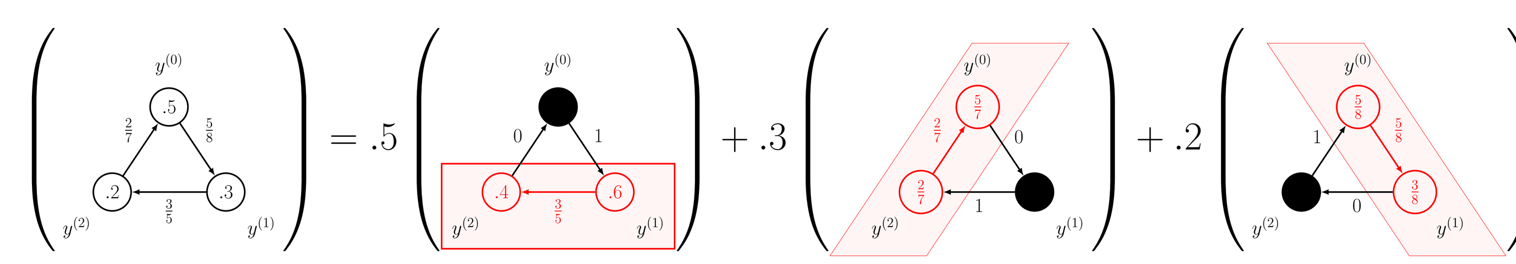
Ranking $x^{(i)}$:



X	$y^{(0)}$	$y^{(1)}$	$y^{(2)}$
$x^{(i)}$	1	ε	ε^2

A table can be decomposed (recursively) into populations of voters.

X	$y^{(0)}$	$y^{(1)}$	$y^{(2)}$
$x^{(i)}$	5	3	2



This decomposes into sub-populations according to their first choice. The red boxes must then be recursively decomposed using the probabilities inside the nodes.

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